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MATHEMATICAL MODELS FOR STATISTICAL ANALYSIS AND CONTROL OF SHEET GLASS ANNEALING

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A method and application results are described for using mathematical models in statistical analysis and control of the annealing regime of a glass band transported inside a tunnel furnace. The results of a computational experiment estimating the efficiency of the algorithms of the annealing regime control are discussed. The possibility for reducing waste and improving the quality of polished glass annealing is demonstrated.

The introduction of the ISO-9002 quality control system in the production of sheet glass implies the use of statistical methods in controlling the process of polished glass production. We will consider the methods and results of the application of mathematical models for analysis and control of the annealing conditions of a moving glass band in a tunnel furnace.

The method consists of the following stages:

– analysis of the annealing process with the aim of choosing an impulse for evaluation of the quality of annealing and refining the determining factors;

– development of "regime – quality" mathematical models describing the dependence of the glass quality parameters on the annealing temperature regime and the monitored variable input parameters;

– development of an algorithm for adjusting the temperature regime depending on the geometrical dimensions of the band and the line capacity, taking into account the controlled disturbances.

The performance of technological line 1 of an annealing furnace produced by the KNUD company was analyzed. The line capacity is 420 tons/day, the glass band width up to 3300 mm, and the band thickness up to 6 mm. For one month the annealing regime data were registered every 4 h and the glass quality parameters were determined.

The quality of annealing was estimated based on the residual stress value, the flat band curvature, and the percent of glass waste after annealing.

The residual stresses in glass were monitored at five different points across the band width. The measurement data are correlated, and the correlation coefficient is 0.83 – 0.95. This made it possible to estimate the residual stress based on the measurement of stresses in the middle of the band. To improve the accuracy, the residual stress was calculated as the arithmetic mean of five measurement results.

The curvature of the manufactured glass was monitored in samples taken from the left and from the right sides of the glass band. The lateral and longitudinal curvatures of the lower and the upper band surfaces were measured. The measurement results correlate, and the correlation coefficient ranges from 0.60 to 0.89. The generalized parameter of the band curvature after annealing was calculated as the arithmetic mean of 8 sample measurements.

The daily glass waste after annealing was calculated based on the operating time of the glass-crushing machine considering its efficiency.

To refine the list of developed models, the closeness of the correlation between the indicators of the produced glass quality was determined. The matrix of pairwise correlation coefficients is given in Table 1.

The data in Table 1 point to a significant correlation existing between the parameters of glass annealing quality. The negative correlation between the glass waste and the residual stress is evidence of a substantial amount of waste

TABLE 1

Glass annealing parameters	Residual stress	Curvature	Glass waste
Residual stress	1	– 0.48	– 0.32
Curvature	– 0.48	1	0.25
Glass waste	– 0.32	0.25	1

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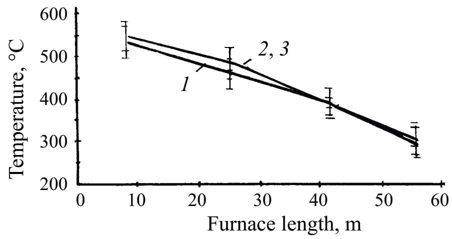

Fig. 1. Averaged curves of annealing temperature variations at different sections across the glass band: *1*) left edge; *2*) center; *3*) right edge.

generated in producing thin glass, and the positive correlation between the waste and the curvature indicates an increase in the glass waste resulting from high-curvature glasses. The negative correlation coefficient between the curvature and the residual stress reflects the greater curvature in thin glass, which have lower residual stresses. The signs of the correlation coefficients correctly characterize the relationships between the individual parameters of glass annealing quality. As the coefficients of correlation between individual quality parameters are low in modulus, they served as the basis for developing all models describing the quality of annealing.

Depending on the geometrical dimensions and the chemical compositions of glass, a specific optimum annealing regime is selected for each type of glass. The present study considers the case where the chemical composition of glass is relatively stable, and its variations are neglected.

The annealing regime depends on the temperature variations in the tunnel furnace and the velocity of the band motion. The temperature inside the annealing furnace is monitored in zones *A*, *B*, *C*, and *D* along the tunnel. The temperature variations in each zone are registered at five points across the band width. To choose an impulse characterizing the annealing temperature conditions, the temperature variations along the furnace axis measured at various points across the band were subjected to statistical analysis (Fig. 1).

A confidence interval for the mean temperature in different zones was constructed for each curve. The averaged curves of temperature variations along the annealing furnace at different cross-sections across the glass band fall within the overlapping confidence intervals. This makes it possible to describe the variations in the mean temperature by a linear dependence of the type

$$\theta = a_0 - a_1 x, \tag{1}$$

where a_0 is the free coefficient of the equation; a_1 is the coefficient of the straight line slope; x is the coordinate of the position of the thermocouple in different furnace zones along the longitudinal axis.

TABLE 2

Parameter	Residual stress	Curvature	Glass waste
a_0	0.03	0.13	0.09
a_1	– 0.27	0.07	– 0.02

Formally the free coefficient corresponds to the glass band temperature at the entrance to the annealing furnace ($x = 0$), which should depend on the temperature of the free end of the glass band at the exit from the float tank. The closeness of the correlation between a_0 and the temperature of the exit end of the band is estimated by a correlation coefficient equal to 0.07, which makes it possible to neglect this dependence. The coefficient of the straight line slope characterizes the intensity of temperature variation along the longitudinal axis of the annealing furnace, which determines the process of annealing of the moving glass band. The effect of a_0 and a_1 on the quality of annealing is characterized by the correlation coefficients shown in Table 2.

As can be seen, a significant correlation exists only between a_0 and the glass curvature, as well as between a_1 and residual stress. The effect of the coefficients on other parameters can be neglected due to its low significance.

To select the factors affecting glass annealing, the following monitored variables that correlate with the glass quality parameters were considered: the temperature of tin in the float tank at span 15, the temperature of the glass band end at the exit from the float tank, the glass density, the production line efficiency, and the thickness and output rate of the produced glass band. Due to the analytic dependence of the efficiency on the band parameters and output rate, the efficiency was excluded from the number of analyzed factors. The glass thickness and the output rate were left for analysis. The temperature of the exit end of the band significantly depends on (correlates with) the temperature of the tin at span 15. The pair correlation coefficient for these temperatures is 0.72, which makes it possible to consider only one temperature, i.e., the temperature of the end of the band at the exit from the float tank.

Thus, five factors were selected for the construction of regression models: the temperature of the glass band end at the exit from the float tank, the glass density, the band thickness, the output rate, and the parameters characterizing the temperature distribution along the annealing furnace (a_0 and a_1).

In developing regression models of the annealing process, the conditions of glass melting and the glass band formation over the tin melt, as well as the chemical composition of the glass, were assumed to be constant. These assumptions simplify the construction of models but affect their precision. The justifiability of these assumptions was substantiated by subsequent studies.

The regression models were constructed from a sampling of 180 experiments reflecting the annealing furnace performance during one month. The structure of the models was refined using the procedure of consecutive regression analysis. At first, all the above listed variables were included in the original structure, and then the insignificant variables were consecutively eliminated using the Student t-criterion. The elimination continued, while the multiple correlation coeffi-

cient kept growing and the residual dispersion of the model kept decreasing. As a result, the following regression models with estimates were obtained:

– residual stresses in glass (μm/mm):

$$\sigma = b_0 + b_1\,\delta + b_2\,v - b_5\,a_1, \quad R^2 = 0.86;$$

– the glass band curvature (mm/mm):

$$C = -\,b_1\,\delta + b_3\,\theta_{\text{exit}} - b_4\,\rho, \quad R^2 = 0.87;$$

– glass waste (%):

$$W = -\,b_1\,\delta + b_3\,\theta_{\text{exit}} - b_4\,\rho + b_5\,a_1, \quad R^2 = 0.74,$$

where $b_0 - b_5$ are the regression coefficients; δ is the band thickness, mm; v is the band velocity, m/min; R^2 is the multiple coefficient of linear correlation; a_1 is the parameter of model (1) of annealing temperature, K/m; θ_{exit} is the temperature of the glass band end at the exit from the float tank, °C; ρ is the glass density, g/cm^3.

Let us consider the efficiency of using the developed models for statistical analysis and control of the glass band annealing process. The matrix of connectivity of the exit parameters of annealing with the annealing temperature conditions, the produced band parameters, and the glass density is shown in Table 3.

Among the specified factors, the intensity of temperature variation along the axis of the annealing furnace was chosen as the controlling factor. As the chilling intensity grows, the waste at the exit from the furnace decreases. At the same time, the residual stress in the glass increases. The glass band curvature is virtually not affected by variations in the chilling intensity. The other factors represent monitored variables and cannot be controlled in the course of annealing. Thus, the glass band thickness, the output rate, and the band temperature at the exit from the float tank are determined by the glass-molding technology and depend on the geometrical dimensions of the band and the efficiency of the production line. The glass density depends on the batch composition and the glass melting process. Thus, one can only control the annealing process through varying the intensity of chilling the moving glass band.

Considering the above, the annealing process control criterion can be written as a penalty function:

$$F2 = \lambda_1 \left| \min(0.25 - \Delta\text{ma1}, 0) \right| + \lambda_2 \left| \min(\sigma_{\text{adm}} - \sigma, 0) \right| + \lambda_3 \left| \min(0.025 - C, 0) \right| + \lambda_4 W, \quad (2)$$

$$\sigma_{\text{adm}} = 5 + 2.5(\delta - 2),$$

where $\lambda_1 - \lambda_4$ are the penalty coefficients; Δma1 is the daily variation in the intensity of glass band chilling in the annealing furnace; σ_{adm} and σ are, respectively, the admissible and the actual residual stress in the glass band.

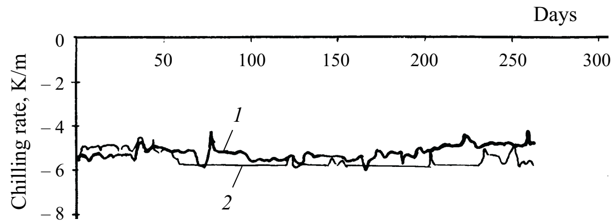

Fig. 2. Conditions of glass annealing on LPS-1 line: *1*) actual values; *2*) model values.

TABLE 3

Glass annealing parameters	Determining factors: band thickness	band velocity	chilling intensity	temperature of the band exit end	glass density
Glass waste	Inverse dependence	–	Linear dependence		Inverse dependence
Residual stress	Linear dependence		Inverse dependence	–	–
Curvature	Inverse dependence	–	–	Linear dependence	Inverse dependence

The first summand in the penalty function (2) limits the daily variation in the chilling intensity of the glass band inside the annealing furnace, and the second one limits the value of the admissible residual stress in the glass, which depends on the produced glass thickness. The third summand is related to the glass band curvature, which should not exceed 0.025. The final summand is proportional to the glass waste value.

The penalty coefficients were selected experimentally: $\lambda_1 = 100$, $\lambda_2 = 3$, $\lambda_3 = 1$, $\lambda_4 = 5$. Simulating modeling of the algorithm of the glass annealing control was carried out using the selected penalty coefficients on the LPS-1 production line at the Borskii Glass Factory between January 1 and December 31, 2000.

The annealing temperature regime can be controlled using the residual stress value. To accomplish this, the penalty coefficients λ_3 and λ_4 in expression (2) are taken equal to zero. In adjusting the annealing regime, the temperature of the exit end of the glass band did not vary and was equal to the actual temperature. The intensity of the temperature variation along the annealing furnace axis is shown in Fig. 2. It can be seen that in statistical controlling, the temperature inside the annealing furnace in producing 3 – 5 mm glass should be maintained virtually invariable. In the case of manual control, a lower intensity of glass chilling intensity was maintained.

The simulation modeling of the algorithm of statistical control of residual stresses in glass revealed the possibility of

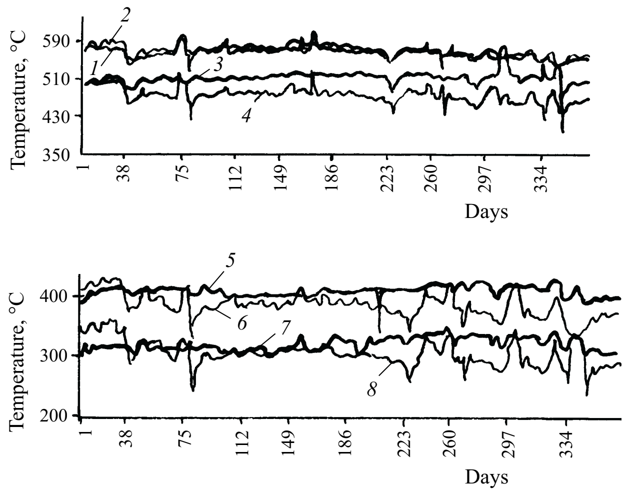

Fig. 3. Temperature in the annealing furnace: *1* and *2*) actual and model temperature in zone *A*; *3* and *4*) actual and model temperature in zone *B*; *5* and *6*) actual and model temperature in zone *C*; *7* and *8*) actual and model temperature in zone *D*.

decreasing the average annual value of residual stresses in glass from 8.98 to 8.38 μm/mm, which is equal to 6.7%. In this case the mean quadratic deviation of residual stresses in produced glass remains virtually constant: 2.31 and 2.06 μm/mm.

The problem of controlling the annealing furnace operation can also be stated as the selection of a regime ensuring the smallest amount of glass waste while satisfying the boundaries imposed on the admissible residual stress value and the glass band curvature. In this case, penalty function (2) is used.

The variations in the annealing temperature are shown in Fig. 3. The model temperature in zone *A* virtually coincides with the temperature registered in the manual control of the annealing process, whereas the temperature in zone *B* is slightly lower. The temperatures in zones *C* and *D* differ insignificantly from the temperatures registered in the manual handling of the process. Only in the second half of the year were these temperatures lower.

The chilling intensity in the model annealing regime differs to some extent (by 9.3%) from the manual control regime. A faster chilling was observed in the second half of the year. The chilling regime in modeling is more stable, and the mean quadratic deviation of chilling intensity is by 9% lower.

TABLE 4

Date	Actual residual stresses		Model residual stresses	
	mean monthly value	mean quadrati deviation	mean monthly value	mean quadratic deviation
23.01.00	10.66	1.68	10.09	1.60
23.02.00	7.99	1.91	7.14	2.00
24.03.00	9.22	1.75	8.77	1.34
23.04.00	9.47	2.19	7.79	1.60
23.05.00	9.57	2.69	8.89	2.44
22.06.00	7.76	1.74	7.34	1.72
21.07.00	8.89	1.88	6.67	1.13
21.08.00	8.65	2.29	8.16	1.57
20.09.00	8.63	2.33	8.81	1.70
20.10.00	8.08	2.26	8.3	1.95
19.11.00	8.76	1.6	8.88	1.51
19.12.00	8.87	2.63	9.14	2.13
Mean annual value	8.88	2.08	8.33	1.73

The comparative results of this control algorithm modeling and the manual control are shown in Table 4.

The use of the annealing control algorithm makes it possible to diminish residual stresses in glass after annealing by 6%, whereas the spread in residual stress values decreases by 16.8%. The curvature of the produced glass band virtually does not change in transition from the manual annealing control to annealing using the control algorithm. The mean curvature is equal to 0.0150 and 0.0156 mm/mm, respectively.

The average daily duration of the operation of the glass-crushing machine in the manually controlled annealing process was 3.45% of the day duration with the mean quadratic deviation equal to 2.7%. In glass annealing using the control algorithm, the average daily operation of the glass-crushing machine was shortened to 2.69%. This yielded an increase by 0.78% in the glass output on the LPS-1 line, which is equivalent to an additional 1169.9 tons of glass (three days of LPS-1 line operation).

The performed studies confirmed the possibility of reducing the glass waste generated on the LPS-1 line and improving the glass quality by using the developed mathematical models and control algorithms.